

Are we made of aliens?

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Contents

1	Introduction	1
2	The technological singularity	2
3	Time and distance scales	2
3.1	Physics at the Planck scale	3
3.2	Constraints on a discrete theory	3
3.3	Computation at the Planck scale	4
4	Migrating to the subquantum level	4
4.1	Developing subquantum science	4
4.2	Populating the universe	5
4.3	Communicating with aliens	6
5	What are the aliens doing?	6
5.1	Consciousness	7
5.2	Mathematical incompleteness and evolution	8
5.3	Expanding mathematics	10
5.4	Consciousness and structure	10
A	Appendix of quotes	11
A.1	Gerard 't Hooft	11
A.2	Richard Feynman	11
	References	12

1 Introduction

The Fermi paradox asks: where are the aliens? There are more than 200 billion stars in our galaxy with an estimated 500 million planets in the habitable zone where earth like life might evolve. This suggests our galaxy may harbor many civilizations, far older and more advanced than ours. Why have we not, so far, seen any unmistakable sign of them? Many answers have been proposed to this paradox.

This speculative article considers the possibility that aliens exist at a subquantum level everywhere including within us, but we are not able to communicate with them. It is based on the time and distance scales of our universe and the possibility that we may be approaching a technological singularity. This is a historical transition where technology reaches and then takes us far beyond human intelligence and creativity. As a consequence we cannot see beyond the singularity.

I let my imagination go wild in this while limiting speculation to what is consistent with what we know, but goes far beyond it. Please take it in that spirit.

2 The technological singularity

Vernor Vinge coined the term singularity[19] as an analogue to black holes where it is impossible to see beyond the event horizon. He wrote that we would eventually be able to construct super intelligent beings who would create a world unimaginable to us. Vinge argued that the singularity might occur in any of four ways.

1. We might develop intelligent computer hardware and software.
2. Computer networks and their users may collectively “wake up” as an intelligent being.
3. Computer enhanced humans with direct intimate interfaces might become super intelligent beings.
4. Biological science might discover ways to enhance biological intelligence.

Ray Kurzweil helped to popularize the term with the *Singularity is Near*[13]. Recognizing that we are far from a comprehensive understanding of human intelligence, his expectations are based in part on our evolving ability to scan the living brain. Kurzweil thought this technology will evolve to the point that we can make a functioning copy of an individual. This would allow a form of immortality where we exist not just as a particular being but also as a super program and program state that can be preserved and duplicated indefinitely. It is this possibility, combined with the time and distance scales of the universe, that suggests that this planet and all it contains may be made of aliens.

3 Time and distance scales

Dramatically different physical laws apply at different time and distance scales. General relativity dominates at the scale of the universe. Classical physics largely explains the world of direct human experience. Quantum mechanics is needed to design computer chips and understand molecules and atoms. At the Planck time and distance scales, quantum mechanics and general relativity are inconsistent and there is no accepted physical theory.

Table 1 lists these scales. On the logarithmic scale of the exponents in the table, biological life is on the high side with more room below than above. Computer circuits are approaching the atomic domain of quantum mechanics that is a theoretical limit for existing technology. However there is vast distance between the atomic scales and the Planck scales.

Size	Meters	Time	Seconds
observable universe	8.8×10^{26}	since big bang	4.4×10^{17}
neuron body	1.0×10^{-5}	chemical synapse delay	2.0×10^{-3}
2011 chip feature	2.2×10^{-8}	2011 chip clock cycle	3.3×10^{-10}
silicon atomic diameter	2.2×10^{-10}	1/max Si spectral frequency	1.7×10^{-18}
Planck distance	1.6×10^{-35}	Planck time	5.4×10^{-44}

Table 1: Time and distance scales of the universe

3.1 Physics at the Planck scale

String theory is the leading contender to reconcile quantum mechanics and general relativity at the Planck scales. String theory is a 40 year old work in progress with many varieties. It has not made testable predictions that differ from quantum mechanics[17, 20].

Both string theory and quantum mechanics use continuous models for a physical system that can be fully specified by a finite amount of information. In quantum mechanics continuous models describe the evolution in time of a wave function that gives the probability of observing a particular physical state. The physical state is not defined between observations and there are only a finite number of possible physical states that may be observed. This suggests that there might be a fully discrete model of physical reality. A discrete model can be constructed by restricting a finite spatial-temporal region, to a finite number of physical locations or points. These points are directly connected topologically. There is no intervening space or time. Space and time are derived from this topology and the laws that define how the state of these points change. The state can be encoded with finite symbols such as the integers.

Near the end of his life, a similar idea seemed plausible, if not likely, to Einstein.

I consider it quite possible that physics cannot be based on the field concept, i. e. on continuous structures. In that case *nothing* remains of my entire castle in the air gravitation theory included, [and of] the rest of modern physics[14, p 467].

Other physicists have speculated about this. For related examples see Appendix A. I have long been intrigued by this idea and the observation that the wave equation occurs throughout physics. In addition to the electromagnetic spectrum (visible light, x-rays, radio waves, etc.) all matter forms waves that define the probability of observing a particle at a particular point in space and time. The classical wave equation is used to model this probability density for a single particle with no rest mass such as the photon.

3.2 Constraints on a discrete theory

Any discrete theory that approximates the wave equation must have two important properties.

- The fundamental equations cannot be exactly linear.
- The maximum rate at which physical effects can propagate must be greater, perhaps much greater, than the velocity of wave propagation i. e. the velocity of light.

See *Emergent Properties of Discretized Wave Equations*[5] for the derivation of these two results. It is possible that recent experiments that seem to show that some neutrinos travel faster than light[1] are the first experimental glimpse into the faster than light phenomena of a discrete universe of the sort that concerned Einstein. It is important to remain skeptical. Even if the experimental results are decisively confirmed, they may not be attributable to discrete space-time.

3.3 Computation at the Planck scale

Regardless of whether or not a discrete theory holds at the Planck scale, new physics must hold there. If this physics can be used to build computers, they may be immensely faster and more powerful than anything conceivable today. In both time and space the Planck scale is well over 10^{-20} the size of the atomic scale which represents a limit of existing computer technology.

The new physics starts well below the scale of atoms, but not necessarily close to the Planck scale. The speed up we assume is a guess. However, if computers can be built at something approaching the Planck scale, they might be 10^{20} or more faster and more complex than today's computers. A 10^{20} speed up in time means one can do in 1 second what would take 3×10^{12} years. This is over 200 times the age of the universe.

4 Migrating to the subquantum level

Assume computers can be built at something approaching the Planck scale and human thought, imagination and consciousness can be transferred to sufficiently powerful computers, as Kurzweil and others believe possible. There will be enormous incentive to migrate to the subquantum level, where one can do in one second what could take 200 times the age of the universe. Such beings would see no point in communicating with us as long as we are ignorant of their technology.

4.1 Developing subquantum science

String theory, if it is true, probably does not lead to new structures that can be used to build computers at a subquantum level. The fundamental particles are those of quantum mechanics. They are very small compared to the size of atoms but not point like as quantum mechanics currently models them. If there is enormous new structure that is only detectable at the subquantum level, then string theory is probably going down a dead end.

As a graduate student I briefly worked on developing a discrete theory that might be relevant to physics. It is easy to simulate such theories, but not at anything approaching the scale that is physically observable. I came to suspect that a discrete physical theory, if it holds, can only be developed by experimenters and theorists feeding each other. This is the way quantum mechanics was developed.

There may be a long research and development phase, once we start seeing some experimental effects that existing physics cannot explain. Perhaps the measurement of superluminal neutrinos at CERN[1] will be confirmed and mark the beginning of this process. It

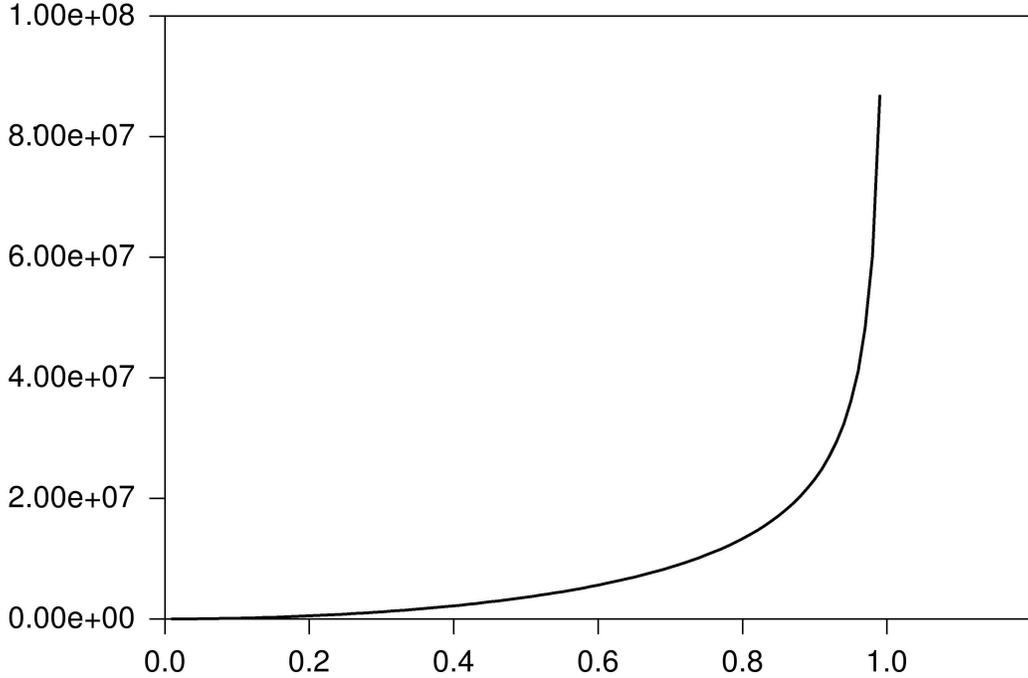


Figure 1: Kilowatt hours to accelerate 1 gram to a fraction (up to .99) of the speed of light

may take a new type of scientific investigation where engineers repeatedly design computers based on new physics and those computers repeatedly make it possible for both theorists and experimental physicists to extend physics. If, during this time, we are able to duplicate human intelligence in machines, the process of physical discovery will proceed at a rapidly accelerating rate.

4.2 Populating the universe

Interstellar space travel is a staple of science fiction and an enormously difficult problem. Figure 1 gives the energy in kilowatt hours to accelerate 1 gram to a fraction of the speed of light. The figure is a plot of Equation 1 with $C = 29979245800$, the speed of light in centimeters/second. f is the velocity as a fraction of the speed of light. m is the rest mass or 1 gram for the plot. The factor, 3.6×10^{13} , is the number of ergs in a kilowatt hour.

$$E = \frac{m(fC)^2}{2 \times 3.6 \times 10^{13} \sqrt{1 - f^2}} \quad (1)$$

The energy required for interstellar travel is far less of an obstacle for subquantum beings. The energy required to accelerate 1 gram to half the speed of light is under 3.61 million kilowatt hours. Of course the actual energy requirements would be much greater. The fuel needed to create this energy must be accelerated to various velocities before it is used and their must be a space ship that uses the fuel to accelerate. Finally fuel is needed to maneuver and perhaps to help to slow down the space ship when it approaches a destination. However

one gram might be an adequate payload for interstellar travel of subquantum beings. There is no need to send sentient beings. One only needs the technology to create such beings when one has located a suitable home.

If the laws of physics permit subquantum computers on anything approaching the Planck scales, it is possible that subquantum beings are now or already have populated the galaxy, carrying out Kurzweil's speculation that intelligent beings may eventually turn much of the mass of the galaxy into sentient intelligence. The might be accomplished with no detectible effects for human observers as long as we lack subquantum technology.

4.3 Communicating with aliens

Science fiction often portrays advanced civilizations building space stations. We have stated to do so at what many believe is an enormous waste of resources. The physicist, Freeman Dyson, has speculated that huge structures may be deployed to capture most or all the energy radiating from a star[8]. Yet, in many ways, we are moving in the opposite direction.

Computers are getting smaller, more powerful and ubiquitous. The most powerful computer is no longer at a single location. It is fragments of the Internet working collectively to solve problems. Cellular phone and data networks improve local communication with relatively weak radio signals. Long distance and area wide broadcasting may be rendered largely obsolete by the Internet and cellular networks. Advanced civilizations may become increasingly inconspicuous as their technology evolves until they reach the subquantum level and become undetectable without their technology. The transition between the technology that may be detectible at installer distances and subquantum technology may be at most a few hundred years. That may be why there are no detectible signs of aliens, even if they are everywhere in our solar system.

If there are beings operating at something close to the Planck time and distance scales, they would have little interest in communicating with us. They could do so but it would be so slow as to be meaningless to them. They may well decide that is best to have no observable impact on the macroscopic level of human experience. They will have plenty of time to deal with whatever happens at the macroscopic level. Even an atomic explosion will be a leisurely paced event for them. There might be a Star Trek like Prime Directive preventing communication except with civilizations that are starting to develop subquantum technology. Where are the aliens? Perhaps everywhere.

5 What are the aliens doing?

Why populate the universe? What is there to do? I suggested that we cannot see beyond the technological singularity, but I think this is only partially true. There are two ideas that give insight into what any sufficiently advanced civilization might do. One concerns the nature of consciousness and the other is based on Gödel's Incompleteness Theorems. The two of them guarantee that there will always be things to do and motivation for doing them.

5.1 Consciousness

Bertrand Russell at the end of *The Analysis of Matter* wrote the following.

As regards the world in general, both physical and mental, everything that we know of its intrinsic character is derived from the mental side, and almost everything that we know of its causal laws is derived from the physical side. But from the standpoint of philosophy the distinction between physical and mental is superficial and unreal[16, p. 402].

If “intrinsic character”, as far as we know, exists only in conscious mental experience, what is the intrinsic nature of inanimate matter? It cannot be the ancient earth, air, fire and water nor the billiard ball like particles of classical physics. Today physics is entirely mathematical and abstract.

I have proposed an answer that I call the Totality Axiom. *Immediate experience in some form is the essence and totality of the existence of physical structure and structure is the only aspect of existence that can be communicated.*[2]. This is a modern version of the ancient idea of panpsychism. The Totality Axiom is not meant to imply that inanimate objects or the world as a whole has a mind, conscious intention or purpose. Those require the evolution of a complex brain.

The idea that only structure can be communicated requires clarification. We often assume that describing an experience can evoke a similar experience in others. If so, that is their conscious experience and not a direct communication of our consciousness. If we say the cloud is orange the communication works if orange refers to similar experiences between two people. It cannot evoke the correct image in someone who was color blind at birth.

The Totality Axiom stems in part from a recognition that there is nothing special about the matter that makes up the human body including the brain. Matter, that is the source of human conscious experience, is special only because of its direct connection to memory and language. We can report aspects of what that matter experiences. The body is more complex than the fragment we are conscious of. All of it may be aware in some sense.

Some of the unconscious parts of the brain have their own memory. For example there are people with brain injuries that prevent them from committing conscious experience to long term memory. They forget all experience (after the injury) in about half an hour. Yet they can still learn and retain new skills, but without the least memory of where those skills came from.

Our conscious experience embeds some of the structure of our brain and nervous system. For example, optical illusions can usually be explained by the physiology of the eye and its neural connections to and in the brain. It is possible that immediate experience in some form may embed all the structure of our brain and of our universe. Human conscious experience extends from enormously complex states to a single point in the visual field. Similar levels of consciousness may exist throughout the brain and body. We are never consciously aware of some of this experience and others we are sometimes aware of. For example the awareness in our left big toe comes into consciousness, but not into existence, when we start paying attention to it. Our big left toe felt like something 10 minutes ago whether or not we were consciously aware of it.

The Totality Axiom makes the simplest possible assumption: matter is immediate experience that fully reflects its physical structure. Structured immediate experience exists at every level of structural complexity and there is no need to assume the existence of anything else. We use internal conscious experience, such as a visual image, to represent the structure of what occurs externally.

The Totality Axiom, combined with physical and biological evolution, suggest that the universe is the creative evolution of consciousness and nothing but the creative evolution of consciousness. A recent New York Times article, “The Mind of a Rock”[11], gives a brief overview of related contemporary ideas on the universality of consciousness. There is an extensive discussion of this idea in my book[2].

Reproducing molecules have evolved into the depth and richness of human consciousness. There is no reason to think that this is a limit of what is possible or that there is a limit. The evolution of complex physical structures leads to enormously deep and rich conscious experience that can be pleasurable or painful and sometimes both. This evolution has been accomplished in part through ever expanding diversity. For example sexual reproduction evolved, despite the complexities it adds to reproduction, because of the diversity it leads to. Yet most spiritual traditions strive for the ultimate rather than the fruits of expanding diversity. They seek union with God, Buddhist enlightenment or some other ultimate state of being. Mathematics, the Totality Axiom, and conscious experience, suggest that any single path approach to the evolution of consciousness leads to a form of stagnation described below as a Gödel limit. However, if we evolve with ever expanding diversity, then whatever ecstatic wondrous experience any being ever has is the merest hint of a shadow of what can be and this will always be the case. There is no final destination. There is only a never ending journey of divergent discovery of ever expanding possibilities and new and richer experience.

5.2 Mathematical incompleteness and evolution

The evolution of consciousness seems to involve complex self referencing structures or feedback loops that allow complex reactions. Flight or fight is a first order reaction to a threat that may not be helpful. For example, a threat to one’s livelihood at work may require a more subtle and complex response.

The richer and more complex the self referencing feedback structures an organism has, the richer their potential for both subtle reactions and deep conscious experience. The human mind is unlikely to embody the limit of this richness and depth. We know it does not have a structural or mathematical limit. It is self referencing structures that allowed Kurt Gödel to prove that any consistent mathematical system of sufficient strength must be incomplete. In the process he showed how self referencing can always be expanded.

One of the implications of Gödel’s work is that any single path expansion of mathematics must lead to a Gödel limit. This occurs when mathematics can always be expanded, but all the expansions that will ever be invented are implied by a single finite axiom that will never be explored. The only way around this is with ever expanding diversity as seems to occur in biological evolution.

Gödel’s proofs were ingenious and complex. They require a good deal of technical mathematics. A simpler related result is the halting problem for a computer that is powerful enough to simulate any possible computer. In the absence of a rigorous definition of “com-

puter”, this is a philosophical statement. The Church-Turing thesis gives a definition of computable or effective algorithm. This thesis is almost universally accepted. Many attempts to define computable or algorithm have led to the same set of computable functions and thus equivalent definitions. The first of these, that was based on a computer model, is called a Universal Turing Machine. Every computer designed today is equivalent to a Universal Turing Machine with the caveat that the mathematical version must have access to unlimited memory and run error free forever.

Gödel’s proof focused on proving the consistency and completeness of formal mathematical systems. We will focus on effective formal systems. These can be defined with a computer program that outputs all the theorems derivable in the system. In 1900 Hilbert proposed developing a complete effective formal system that could decide every arithmetical question and was consistent. Gödel’s work proved this was impossible.

Gödel proved a formal system that was powerful enough to embed the primitive recursive functions (and thus a Universal Turing Machine) must be inconsistent or incomplete. It could not prove its own consistency[9]. The consistency of any effective formal system is equivalent to the halting problem for a computer program that is straight forward to construct. The recursive unsolvability of the halting problem proves that there can be no computer program that correctly decides the consistency of every effective formal system. Thus no effective formal system is complete in deciding objective mathematical questions.

Gödel’s proofs depend on showing that every statement in a formal system could be mapped into a unique integer. This is called Gödel numbering. Every computer program has a unique binary (and thus numeric) code in a computer’s memory. This can be thought of as the Gödel numbering of programs. The halting problem is to construct a computer program, H , with a single input that is the Gödel number of another program, C . For every input, this program must run for a finite time and then output a 1 if the input program will halt and 0 otherwise. This output is the value of $H(C)$. The first step is to show how to modify a computer program that solves the halting problem so it will solve the self halting problem.

The self halting problem asks will a program D with a single parameter halt if it is run with its own Gödel number as input. Given D one can construct a program, E , which halts if and only if $D(D)$ does. One can modify H to construct S that solves the self halting problem by adding to it the construction of E from D . E is constructed to contain both D as a program and D as a Gödel number. E with no parameter must do exactly what $D(D)$ does. Then $H(E)$ and thus $S(D)$ will determine if $D(D)$ halts.

The final step is to show that a solution to the self halting problem leads to a contradiction. Modify $S(C)$ to construct $M(C)$ which first solves the self halting problem for C and then loops forever if C halts or halts if C never halts. What does $M(M)$ do? If it halts it runs forever and vice versa. This contradiction implies that there can be no solution to the self halting problem and thus no solution to the halting problem.

It is the ability of a computer program to reproduce the computation of any other computer that makes it impossible to construct a computable solution of the halting problem. Gödel’s proof depended on showing that the formal system called first order arithmetic could model itself.

5.3 Expanding mathematics

Although the halting problem is not solvable for all computer programs there is no particular program for which it is not solvable. There are infinite sequences of effective formal systems such that some element of the sequence decides every halting problem and no element of the sequence makes an incorrect decision. Of course, for some problems in every system, no decision is made. These sequences are not recursively enumerable, i. e. they cannot all be output by a single computer program. The only way to explore every member of such a sequence is with a divergent approach to mathematical truth where an ever increasing number of alternatives are explored.

An effective formal system can be expanded by adding the axiom that the system is consistent. This is a new system. The axiom that asserts consistency only applies to the original system without this axiom. An infinite sequence of axioms like this each of which refers to the previous system can be assumed. These are recursively enumerable axioms and all of them can be added as a single axiom. This whole system can be asserted to be consistent. In this way a recursively enumerable infinite sequence of recursively enumerable sequences of such axioms can be constructed. This approach to extending mathematics is generalized in the recursive ordinals[12].

Classes of ordinals can be defined. For example the recursive ordinals are those whose structure can be completely modeled by a computer program. With this definition, in a more rigorous form, one can assert the axiom that the consistency of a formal system can be iterated up to any recursive ordinal.

There is a hierarchy of ordinals beyond the recursive ordinals that are essential for deciding questions about divergent creative processes like biological evolution. One example involves an unbounded universe in which all the laws of physics are computable. The question is: will a species that may have an infinite number of direct descendant species have an infinite chain of descendant species?

For a general discussion of ordinal analysis see “The Art of Ordinal Analysis”[15]. For more about this approach to mathematics see “What is mathematics about?”[3]. For interactive examples of ordinal arithmetic see the ordinal calculator[6].

5.4 Consciousness and structure

Evolution has climbed a hierarchy of structural complexity and consciousness. We can explain much of the structure of consciousness through research on the brain and neural networks. This understanding is increasing at an accelerating rate as our technical ability to scan the living brain improves.

Starting with cochlear implants, that give hearing to the deaf, we are using devices with direct connections to the nervous system to correct defects. Cochlear implants are crude replacements for the transducer in the inner ear that converts sound waves to neural signals. They cannot reproduce the subtlety of human hearing. A person who lost their hearing must literally rewire their brain over time to hear through these artificial devices[7].

We can expect continual improvement in devices that connect directly to the nervous system until they reach and then exceed the power and subtlety of the biological structures they replace. Eventually we will be able to replace damaged regions in the brain itself.

As this ability becomes more comprehensive the difference between the animate and the inanimate will fade.

The motivation for moving human consciousness into artificial devices may come to focus less on immortality and more on expanding experience in ways that are not possible with existing biological mechanisms.

The unbounded expansion of mathematics can only be explored by an ever expanding divergent process. If the structures discovered in this way lead to deeper and richer conscious experience as biological evolution suggests, then there will be a powerful motivation to populate the universe with beings engaged in the divergent process of exploring mathematical truth and the conscious experience that is the essence of all that physically exists[4]. If aliens are here that may explain what they are doing.

We know there is subquantum physics that we do not understand. If the correct model is based on string theory or some other possibilities, the new physics may not lead to subquantum structures of the sort speculated about here. However, if that physics is based on discrete models as Einstein came to suspect, then it probably does lead to subquantum physics on which enormously powerful computers can be built.

A Appendix of quotes

A.1 Gerard 't Hooft

Awarded the 1999 Nobel prize in physics

On the possibility of a local deterministic theory of physics

Quantum mechanics could well relate to micro-physics the same way thermodynamics relates to molecular physics: it is formally correct, but it may well be possible to devise deterministic laws at the micro scale. Why not? The mathematical nature of quantum mechanics does not forbid this, provided that one carefully eliminates the apparent no-go theorems associated to the Bell inequalities. There are ways to re-define particles and fields such that no blatant contradiction arises. One must assume that all macroscopic phenomena, such as particle positions, momenta, spins, and energies, relate to microscopic variables in the same way thermodynamic concepts such as entropy and temperature relate to local, mechanical variables. The outcome of these considerations is that particles and their properties are not, or not entirely, real in the ontological sense. The only realities in this theory are the things that happen at the Planck scale. The things we call particles are chaotic oscillations of these Planck quantities[18].

A.2 Richard Feynman

On complexity in physics

It always bothers me that, according to the laws as we understand them today, it takes a computing machine an infinite number of logical operations to figure

out what goes on in no matter how tiny a region of space, and no matter how tiny a region of time. How can all that be going on in that tiny space? Why should it take an infinite amount of logic to figure out what one tiny piece of space/time is going to do? So I have often made the hypotheses that ultimately physics will not require a mathematical statement, that in the end the machinery will be revealed, and the laws will turn out to be simple, like the checker board with all its apparent complexities[10, p 57].

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